THE DYNAMICS OF $J = \frac{3}{2}^{+}$ $\bar{K}\,\Xi$ AND KN SYSTEMS IN THE 27 FOLD REPRESENTATION[+]

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ABSTRACT

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We study the dynamics of the $I = 1$, $J = \frac{3}{2}^+$ $\bar{K}\,\Xi$ and KN systems by the N/D method. The dynamical singularities of the partial wave amplitude are assumed to arise mainly from the nearby cut (due to Σ and Λ exchange in the crossed channel) and the far left hand cut ($-\infty < s \leq 0$). The contribution of the former is evaluated explicitly in terms of the relevant Yukawa coupling constants and that of the latter, by the method of Balázs, through the introduction of effective range pole terms. We find that for a wide range of choice of the relevant Yukawa coupling constants, there exist self-consistent bound state (or resonant) solutions for both $I = 1$, $\bar{K}\,\Xi$ and KN-systems. Self-consistent solution for the position in case of the former is found to lie in the range 1650 $\sim$ 1870 Mev and that for the latter in the range 1300 $\sim$ 1600 Mev. The overall conclusion is found to be rather insensitive to the choice of the relevant Yukawa coupling constants.

It is discussed inthe Appendix that it appears to be a typical feature of Balázs type procedure for treating the far left hand singularities, that in almost any $J = \frac{3}{2}^+$ Baryon-meson system one would obtain self-consistent bound state or low lying resonant solution, unless the Born terms are very strongly repulsive. Experimental confirmation on the existence or nonexistence of such systems would, thus, have strong implications on the dynamical methods such as for example adopted in the present note.

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I. INTRODUCTION

The discovery of the Ω^--hyperon at about 1680 Mev fits beautifully into the 10-fold representation of SU(3). From the dynamical point of view it occurs as a pole (in this case a bound state) in the $I = 0$, $J = 3/2^+$ - $\bar{K}\,\Xi$ - scattering, which presumably arises through the forces due to Σ and Λ-exchange in the u-channel. Explicit dynamical treatment of the above scattering via the N/D method with the specific purpose of studying whether or not one should expect a bound state (or a resonance) in the above system has been carried out by many authors.[1] One may summarise the results by saying that the qualitative aspects of the existence of Ω^- is perhaps well understood dynamically. One interesting feature that puts the dynamical treatment of Ω^- on a somewhat better footing than those of its predecessors such as Ξ^*[2] and Y_1^*[3] is that to a fairly good approximation Ω^- is essentially a one channel problem; hence the virtues or defects of the dynamical methods may be attributed to sources other than the inherent multichannel problem (For example; to the treatment of the far left hand cut singularities, inelastic effects, etc.). The other feature (which is common to the Ξ^*[2] -problem as well) is that in the method, where one treats the far left hand cut (f.l.h.) singularities by the effective range pole terms[4] (hopefully this is better than just considering the explicit contribution of the Born singularities) the dependence on the relevant Yukawa couplings is not so marked at least for the qualitative aspects of the conclusion. This may, at the outset, also be regarded as a virtue (however see discussion in Sec. IV, and Appendix).

With these remarks we note that there are three distinct systems in the 27-fold representation of SU(3), which also have the characteristic of being essentially single channel problems. They are the $I = 1$, $\bar{K}\,\Xi$ -system, $I = 1$ KN-system and $I = 2$ $\Sigma\,\pi$-system. Analogous to the Ω^- problem we, therefore, study in this note the dynamics of the $I = 1$, $J = 3/2^+$ - $\bar{K}\,\Xi$ and KN-systems with a view to find out whether or not one should expect a bound state (or a resonance) in these systems, if so at what energies and with what residues. The $I = 2$, $\Sigma\,\pi$-system is studied similarly in a separate note.[5]

We find that for a wide range of choice of the relevant Yukawa coupling constants, there exist self-consistent bound state (or resonant) solutions for both $I = 1$ $\bar{K}\,\Xi$ and KN-systems. The results are presented in Sec. III and are discussed in Sec. IV. In the Appendix we show that it appears to be a typical feature of the Balázs procedure for treating the f.l.h. singularities that in almost <u>any $J = 3/2^+$ Baryon-pseudoscalar meson system, one would obtain self-consistent bound state or low lying resonant solution</u>, except when the Born terms are very strongly repulsive. It is discussed in Sec. IV that experimental confirmation on the existence or nonexistence of such systems will, therefore have strong implications on the dynamical methods such as for example adopted in the present note.

II. THE $J = \frac{3}{2}^+$, $I = 1$, $\bar{K}\,\Xi$ AND KN-SYSTEMS

<u>$\bar{K}\,\Xi$-System:</u>

We will first discuss scattering in the $I = 1$, $J = 3/2^+$ - $\bar{K}\,\Xi$-system and mention later what substitutions are needed for the corresponding KN-system. We will follow the same notations and almost the same procedure as in I.[3] The reader is referred to I for details. The singularities of the partial wave amplitude in the unphysical region arise from (i) Λ and Σ-exchange in the u-channel (ii) higher mass exchanges in the u-channel and (iii) Vector-Meson and higher mass exchanges in the t-channel. We neglect (ii) and (iii) in so far as they contribute to the singularities in the unphysical region in the right half s-plane, which, therefore is assumed to arise solely through (i). The contributions of (i), (ii) and (iii) to the singularities in the left half s-plane are represented by effective range pole terms whose positions and residues are determined by the procedure suggested by Balázs.

Writing the partial wave amplitude $g_{1^+}(s)$ as usual in the form $N(s)/D(s)$, where N contains all the unphysical singularities and D the physical right hand cut, we have (using elastic unitarity)

$$D(s) = 1 - \frac{s-s_o}{\pi} \int_{(\bar{K}+\Xi)^2}^{\infty} \frac{q'^3 N(s')}{s'(s'-s)(s'-s_o)} ds' \qquad (1)$$

and

$$N(s) = N_{(f)}(s) + N_{(n)}(s) \qquad (2)$$

where

$$N_f(s) = \frac{1}{\pi} \int_{-\infty}^{o} \frac{\{\mathrm{Im} g_{1^+}(s')\} D(s')}{s' - s} ds' \approx \frac{b_3}{s-s_3} + \frac{b_4}{s-s_4} \qquad (3)$$

b_3 and b_4 are unknown constants independent of s and will be determined by the use of the fixed energy dispersion relation; s_3 and s_4 are determined by drawing the Balázs curves and are found to be:

$$s_3 \simeq -22.6\ m_\pi^2 \quad , \quad s_4 \simeq -625\ m_\pi^2 \qquad (4)$$

$$N_{(n)}(s) = \frac{1}{\pi} \int_{L_1(\Sigma)}^{L_2(\Sigma)} ds' \frac{\mathrm{Im} g_{1^+}^{(\Sigma)}(s')}{s' - s} + \frac{1}{\pi} \int_{L_1^{(\Lambda)}}^{L_2^{(\Lambda)}} ds' \frac{\mathrm{Im} g_{1^+}^{(\Lambda)}(s')}{s' - s} \tag{5}$$

where

$$g_{1^+}^{(Y)}(s) = \frac{g^2_{Y \Xi K}}{32\pi q^4} [\{(W + \Xi)^2 - K^2\} (W + Y - 2\,\Xi) Q_1(a)$$

$$+ \{(W - \Xi)^2 - K^2\} (W + 2\Xi - Y)\, Q_2(a)] \tag{6}$$

$$a = \frac{\{2(\Xi^2 + K^2) - W^2 - N^2\}}{2q^2} + 1 \tag{7}$$

$Q_\ell(x)$ stands for the legendre function of the second kind, $g_{Y\Xi K}$ for the $Y\Xi K$ coupling constant and Y for Σ or Λ. The end points of the cuts are given by

$$L_1(Y) = \frac{(\Xi^2 - K^2)^2}{Y^2} \quad ; \quad L_2(Y) = 2(\Xi^2 + K^2) - Y^2 \tag{8}$$

Thus:

$$L_1(\Lambda) = 91\, m_\pi^2 \quad , \quad L_2(\Lambda) = 139\, m_\pi^2$$

$$L_1(\Sigma) = 79\, m_\pi^2 \quad , \quad L_2(\Sigma) = 129\, m_\pi^2 \tag{9}$$

As mentioned in I, the nearby cut contribution to the N-function (i.e., $N_{(n)}(s)$) is well represented by a two pole formula for s in the physical region. Thus for $s \gtrsim (\bar{K} + \Xi)^2$

$$N_{(n)}(s) \simeq \frac{b_1}{s-s_1} + \frac{b_2}{s-s_2} \quad (s \gtrsim (\bar{K} + \Xi)^2) \tag{10}$$

where by explicit evaluation of $N_{(n)}(s)$ (as in I) for a few values of s, we find

$$s_1 = 95\ m_\pi^2 \quad ; \qquad s_2 = 123\ m_\pi^2 \tag{11}$$

The residues b_1 and b_2 are determined for a given choice of (i) the coupling constants $g^2_{YNK}/4\pi$, (ii) the subtraction point s_o and (iii) the input value of the position $((sR)_{in})$ of the bound state or resonance under examination.

For s below the physical region (i.e., for $s = sM_1$ or sM_2, to be introduced below) $N_n(s)$ is evaluated explicitly by numerical integration.

From Eqs. (1), (2), (3) and (10), the D-function is given by

$$D(s) = 1 - \frac{s-s_o}{\pi} \sum_{i=1}^{4} b_i\ F(s,s_i,s_o) \tag{12}$$

where

$$F(s,s_i,s_o) = \int_{(\bar{K}+\Xi)^2}^{\infty} \frac{(q'^3/s')}{(s'-s)(s'-s_o)(s'-s_i)}\ ds' \tag{13}$$

The F-functions are evaluated numerically by IBM 7094 for various values of the argument s. Thus the partial wave amplitude N(s)/D(s) is determined except for the unknown constants b_3 and b_4. To determine b_3 and b_4 we use fixed s dispersion relation (FSD) in a region where the partial wave expansion is expected to be convergent. The fixed energy dispersion relation for the invariant amplitudes A(s,t,u) and B(s,t,u) are given by

$$A(s,t,u) = \frac{R_\Sigma}{u-m_\Sigma^2} + \frac{R_\Lambda}{u-m_\Lambda^2} + \frac{R_B}{s-s_R} + \frac{1}{\pi}\int dt'(---) + \frac{1}{\pi}\int du'(---) \tag{14}$$

A similar expression holds for B(s,t,u). The third term corresponds to the contribution of the bound state or resonance in the direct channel with unknown position and residue. We drop the contribution from the integral terms on the RHS of Eq. (14) as well as the vector meson exchange pole terms in the t-channel (These can be shown to be small.). Thus our partial wave amplitude approximated by the FSD in the appropriate region is given by

$$g_{1^+}(s) \simeq g_{1^+}^{(\Sigma)}(s) + g_{1^+}^{(\Lambda)}(s) + g_{1^+}^{(\mathrm{Bound})}(s) \tag{15}$$

where

$$g_{1^+}^{(\mathrm{Bound})}(s) = -(\kappa)\left\{\frac{(W+\Xi)^2 - K^2}{(W_R+\Xi)^2 - K^2}\right\}\frac{1}{W-W_B} \tag{16}$$

W_B stands for $s_B^{\frac{1}{2}}$ and denotes the mass of the bound state (or resonance); κ is the corresponding residue.

If we equate the right side of Eq. (15) with N(s)/D(s) at two points sM_1 and sM_2 (chosen appropriately such that both the representation (15) and the approximate representation for N(s)/D(s) outlined above hold at sM_1 and sM_2), we can evaluate b_3 and b_4 in terms of the input values of s_R and κ (called $(s_R)_{in}$ and $(\kappa)_{in}$) for a given choice of the Y Ξ K coupling constants. We choose, subject to the criterion discussed in I,

$$sM_1 = 75\, m_\pi^2, \quad sM_2 = 142\, m_\pi^2 \tag{17}$$

Once b_3 and b_4 are determined, as mentioned above, one can compute the D-function, look for the zero of the real part of the D-function, which will be identified as the output value of s_R ($(s_R)_{out}$) and the corresponding output value of the residue is given by

$$(\mathcal{K})_{out} = -\frac{N[(s_R)_{out}]}{ReD[(s_R)_{out}]}\left(\frac{1}{2(W_R)_{out}}\right) \tag{18}$$

Solutions for s_R and $\mathcal{K}$ are to be regarded as acceptable if their output values are consistent (to within say 5%) with the input values. It is hoped that these solutions, if they exist, will lie within a narrow range and will determine the question of existence of the physical bound state (or resonance).

<u>I = 1, J = 3/2$^+$ - KN-System:</u>

The treatment of I = 1, J = 3/2$^+$ - KN system is very similar to that of the I = 1, J = 3/2$^+$-$\bar{K}\,\Xi$ system. The singularities of the partial wave amplitude of both systems arise from similar exchanges in the crossed channels, such as, for example, Λ and Σ exchange in the u-channel. Thus the treatment of the KN system is simply obtained from that of the $\bar{K}\,\Xi$-system by the following substitutions

$$\bar{K} \to K \quad ; \quad \Xi \to N \ , \quad g_{\Lambda\Xi K} \to g_{\Lambda NK} \quad , \text{and } g_{\Sigma\,\Xi K} \to g_{\Sigma NK} \tag{19}$$

For the $\overline{KN}$ system; essentially due to $(\Xi - N)$ mass difference we find

$$L_1(\Lambda) \simeq 16.6\ m_\pi^2\ ,\ L_2(\Lambda) \simeq 51.6\ m_\pi^2\ ,\ L_1(\Sigma) \simeq 14.5\ m_\pi^2\ ,\ L_2(\Sigma) \simeq 42\ m_\pi^2$$

$$s_1 \simeq 18.8\ m_\pi^2 \quad , \quad s_2 \simeq 39.4\ m_\pi^2 \quad ,$$

$$s_3 \simeq -17\ m_\pi^2 \quad , \quad s_4 \simeq -400\ m_\pi^2 \tag{20}$$

we choose

$$sM_1 = 60\ m_\pi^2 \ , \qquad sM_2 = 80\ m_\pi^2$$

and

$$s_o = sM_1 \tag{21}$$

The matching procedure for KN system and the testing of existence of self-consistent solutions for bound states or resonances are done in the same way as for the $\bar{K}\,\Xi$-system. The results for both systems are summarised in the following section.

III. RESULTS

In the following we first summarise the results for the $J = 3/2^+$, $I = 1$, $\bar{K}\,\Xi$-system and then do the same for the corresponding KN-system.

$\underline{J = 3/2^+,\ I = 1,\ \bar{K}\,\Xi\text{-System}}$:

(1) First of all, irrespective of the choice of the relevant Yukawa coupling constants (confined within a reasonable range) we find that there does exist self-consistent solution for position and residue indicating the existence of bound state (or resonance) in the $I = 1$, $\bar{K}\,\Xi$-system (call the corresponding particles as z^{--}, z^- and z^o).

(2) In the SU(3) limit, $g^2_{\Sigma\,\Xi K}/4\pi \simeq 15$ and for a f/d-ratio $\simeq \frac{1}{3}$; $g^2_{\Lambda\Xi K}/4\pi \simeq 0$. Since it is not clear how well these predictions are obeyed by the physical coupling constants apart from what is a reasonable value for the f/d-ratio, we tried a wide range of values for these coupling constants. We chose $g^2_{\Sigma\,\Xi K}/4\pi = 16, 8, 4, 1, 0$ and independently $g^2_{\Lambda\Xi K}/4\pi = 16, 8, 4, 1, 0$. We find, as in the Ξ^*-problem, the results are rather insensitive to the

choice of the Yukawa coupling constants. Of course, as is expected, there is found to be a gradual increase in the self-consistent value of the residue $\mathcal{K}$, with an increase of the effective[6] coupling constant. Except for this, it is found that there is not a very marked dependence of the position of the bound state (or resonance) on the choice of the coupling constants. In other words, if we impose that the self-consistency in position and residue be good to say 5 $\sim$10%, then for any choice of coupling constants in the above range and for input values of the position in a reasonable range (say $140 \sim 180\ m_\pi^2$); one can pick an input value of $\mathcal{K}$ for which the output values of s_R and $\mathcal{K}$ are consistent with the corresponding input values. The input values of s_R that give rise to self-consistent output values are found to lie within the range

$$s_R \simeq 140 \sim 175\ m_\pi^2 \tag{22}$$

Above[7] this range the self-consistency becomes poorer. Depending upon the choice of the coupling constants the corresponding self-consistent solution for $\mathcal{K}$ is found to lie within the range

$$\mathcal{K} \simeq 8 \sim 16 \tag{23}$$

Of course, if one demands better and better degree of self-consistency (say better than 1% in both residue and position) the self-consistent solution for a given choice of coupling constants get restricted to a narrower region. This is part of the reason of quantitative discrepancy between the results of Kane[8] and that of ours. Qualitatively the results agree. We feel that it is hard to judge a priori how good a self-consistency should one really expect in such an approximate method. So it may not be proper to disregard solutions which are not exactly self-consistent but are so within, say 5 to 10%. The other reason of discrepancy is the neglect of $\Sigma\ \Lambda$-mass difference in Kane's work.

(3) Again as in the Ξ^* problem, with self-consistent value of $\mathcal{K}$ and $g^2/4\pi < 5$ (say), the effective range pole terms denoting the far left hand cut contribution ($N_{(f)}$) are found to be larger than the nearby cut contribution ($N_{(n)}$) by at least an order of magnitude. Similarly, in the fixed energy dispersion relation the Σ, Λ-contribution to g_{1^+} is found to be smaller than that of the bound state term ($g_{1^+}^{(\text{Bound})}$).

<u>$J = 3/2^+$, $I = 1$ - KN-System:</u>

The qualitative aspects of the results (1), (2) and (3) mentioned above also hold for the KN-system. The self-consistent solutions for the position and residue are found to lie in the range

$$\begin{aligned} s_R &\simeq 85 \sim 135\ m_\pi^2 \\ \mathcal{K} &\simeq 7.5 \sim 11 \end{aligned} \tag{24}$$

Above 135 m_π^2 the self-consistency becomes considerably poorer.

Thus one should expect not only a bound state (or resonance) in the $J = 3/2^+$, $I = 0$, $\bar{K}\,\Xi$ (i.e., Ω^-) and $I = 1$, $\bar{K}\,\Xi$-system (i.e., z^{--}, z^-, z^0), but also in the $J = 3/2^+$, K^+p-system. The later two are yet to be found. Their exact locations cannot be predicted too accurately in the present framework. However z^{--} is predicted to lie roughly in the range 1650 $\sim$ 1870 Mev while K^+p bound state (or resonance) is predicted to lie in the range 1300 $\sim$ 1600 Mev.

IV. DISCUSSION

The $I = 1$, $\bar{K}\,\Xi$-system (z^{--}, z^{-}, z^{0}) and the $I = 1$, KN-system ($K^{+}p$, etc.) belong to the 27-fold representation of SU(3). One would expect, on the basis of the present calculation and from the point of view of unitary symmetry, to observe a host of $J = 3/2^{+}$ 27-fold bound states (or resonances) in the Baryon-meson system in addition to the already observed 10-fold representation. So far there is some indication of the existence of only one system which belongs to the 27-fold representation, i.e., the resonance in $\Sigma^{-}\pi^{-}$-system around 1415 Mev.[9] From the experimental point of view the detection of z^{--} ($I = 1$, $Y = -2$), if it were produced in the $K^{-}p$ experiment, would be considerably easier than that of Ω^{-}. Since it has not been detected as yet, one would guess that it lies, if at all, quite a bit higher than Ω^{-}. As regards $K^{+}p$ system, there is already strong experimental indication[10] on the absence of any bound state or resonance in this system. At this stage we only note that if experiments confirm the absence of any bound state or resonance in the $J = 3/2^{+}$, $I = 1$, $\bar{K}\,\Xi$ and/or KN system, one would seriously question the success of the methods and the results in the previous dynamical calculations. These remarks are related to a general feature of the Balázs-type bootstrap procedure, which we note in the following.

We observe that various types of $J = 3/2^{+}$-systems, such as $\overset{*}{N}_{3/2}$ [11], $\overset{*}{Y}_{1}$ [3], $\overset{*}{\Xi}$ [2], Ω^{-} [1], z^{--}, $K^{+}p$ and $\Sigma^{-}\pi^{-}$ [5] etc., subject to the bootstrap procedure as outlined in the present note, have all yielded self-consistent bound state (or resonant) solutions. From this one might guess that perhaps the analysis is not so sensitive to the choice of the system, its strangeness, isospin and SU(3) representation and that it will lead to a self-consistent bound state or resonant solution in any $J = 3/2^{+}$ Baryon (B)-pseudoscalar meson (P)-system, except when the Born terms may be very strongly repulsive.[12] This is a remarkable and somewhat awkward result, if it corresponds to reality, although it appears very unlikely. We show in the Appendix that this situation is typical of at least the Balázs' type bootstrap procedure, in which the Yukawa coupling constants play a role in a rather inconspicuous way. This leads one to wonder about the physical implications of the results in such a scheme. At any rate the most interesting question is: will experiments preserve such a conclusion anyway?

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APPENDIX

A TYPICAL FEATURE OF BALAZS TYPE PROCEDURE

Typically in a problem involving (B+P)-scattering one has: $B \approx 6.5 \sim 10\ m_\pi$, $P \approx 1 \sim 4\ m_\pi$; one is interested in studying the occurrence of bound state or resonance in the range of $(B+P) \pm 50\ m_\pi^2$ (say); thus $(B+P)^2 - 50\ m_\pi^2 < (s_R)_{in} < (B+P)^2 + 50\ m_\pi^2$; sM_1 and sM_2 (the two matching points) are roughly $20 \sim 60\ m_\pi^2$ below threshold. Under these circumstances one can check quite generally that irrespective of the choice of the Yukawa coupling constants (confined within a reasonable range) one has to take in the first place a value of K_{in} which is at least $6 \sim 12$ to have self-consistency in the position (i.e., $(s_R)_{out} = (s_R)_{in}$). For such values of K_{in} it is found by explicit calculations in various systems that the overall conclusion on the existence (and perhaps even the location) is hardly affected by varying the Yukawa coupling constants by more than an order of magnitude ($g^2/4\pi \simeq 0 \sim 16$, say). Furthermore, at least when the Yukawa coupling constants are not very large ($g^2/4\pi \approx 1$, say) one finds that in general $N_{(f)}$ and $g_{1^+}^{(Bound)}$ are bigger than $N_{(n)}$ and $g_{1^+}^{(Born)}$ respectively by an order of magnitude or more. Given this; one is led to ask; what would happen if one had put to start with;

$$\begin{aligned} & b_1 = b_2 = 0 \\ \text{and} \quad & g_{1^+}^{(Born)} = 0 \end{aligned} \tag{A.1}$$

One then needs to solve for b_3 and b_4 from the matching equations given by

$$\frac{b_3/s-s_3 + b_4/s-s_4}{1 - \frac{s-s_o}{\pi} \sum\limits_{i=3}^{4} b_i\ F(s,s_i,s_o)} \approx - K_{in}\ \frac{(W+B)^2 - P^2}{[(W_R)_{in}+B]^2 - P^2}\ \frac{1}{W-(W_R)_{in}} \tag{A.2}$$

The above equality is expected to hold in an appropriate region (see discussion on this in I), within which one chooses the two matching points sM_1 and sM_2. By matching the right and left sides of the above equation, we can evaluate b_3 and b_4 in terms of $\mathcal{K}_{in}$ and $(s_R)_{in}$. One can then compute the D-function, the zero of whose real part gives $(s_R)_{out}$; the corresponding value of $\mathcal{K}_{out}$ is given by (see Eq. (18))

$$\mathcal{K}_{out} \simeq - \frac{b_3/[(s_R)_{out} - s_3] + b_4/[(s_R)_{out} - s_4]}{R_e D'((s_R)_{out})} \frac{1}{2(W_R)_{out}} \qquad (A.3)$$

Choosing B, P and $(s_R)_{in}$ in the range mentioned above, one can now check quite generally the following:

(A) Firstly starting from Eq. (A.2) one can solve for $\mathcal{K}_{in}$, that leads to $(s_R)_{out} = (s_R)_{in}$. It is found that there always exists a value of $\mathcal{K}_{in}$ typically in the range of 8 ~ 12 which yields $(s_R)_{out} = (s_R)_{in}$, chosen in the range mentioned above.

(B) If one next asks what is the value of $\mathcal{K}_{out}$ corresponding to $\mathcal{K}_{in}$ so chosen as to yield $(s_R)_{out} = (s_R)_{in}$ (as mentioned in (A)), one finds (as may have been expected from Eqs. (A.2) and (A.3)); if $(s_R)_{in}$ is chosen in a certain range below the physical threshold (corresponding to a bound state solution), where the fixed energy dispersion relation is expected to hold to the same extent that we used it to determine b_3 and b_4; then $\mathcal{K}_{out}$ is identically equal to $\mathcal{K}_{in}$. This may be seen as follows. For values of $(s_R)_{in}$ as mentioned above we may put;

$$N(s_R)_{out}\Big]_{(s_R)_{out}\to(s_R)_{in}} = \left[\frac{b_3}{(s_R)_{out}-s_3}+\frac{b_4}{(s_R)_{out}-s_4}\right]_{(s_R)_{out}\to(s_R)_{in}}$$

$$\approx -\mathcal{K}_{in}\left\{\frac{[(W_R)_{out}+B]^2-P^2}{[(W_R)_{in}+B]^2-P^2}\qquad\frac{D(s_R)_{out}}{(W_R)_{out}-(W_R)_{in}}\right\}_{(s_R)_{out}\to(s_R)_{in}}$$

$$\simeq -\mathcal{K}_{in}\,(2W_R)_{out}\,\dot{D}(s_R)_{out} \tag{A.4}$$

Using Eq. (A.3) it follows $\mathcal{K}_{out} = \mathcal{K}_{in}$. This would imply that in the Balázs' procedure; in so far as the Born terms and $N_{(n)}$ are much smaller than $g_{1^+}^{(Bound)}$ and $N_{(f)}$ respectively, one is almost guaranteed to obtain at least bound state self-consistent solution for any $J = 3/2^+$[13] (B+P)-system. In some cases (specially when the relevant Yukawa coupling constants are large) the Born terms and $N_{(n)}$ are appreciable. These may make some quantitative difference in the results. However in actual practice, it is found by explicit calculation in various systems (References 1, 2, 5 and present note, etc.) that there do exist very good self-consistent solutions even for a very wide range of values of the Yukawa coupling constants, ($g^2/4\pi$ $0 \sim 20$, say). Thus it appears that even with the inclusion of the Born terms ($g_{1^+}^{(Born)}$ and $N_{(n)}$) the overall qualitative conclusion regarding the existence of self-consistent solution corresponding to either a bound state or a low-lying resonance[14] in every $J = 3/2^+$ (B+P)-system will still be maintained, barring the situation where the Born terms are strongly repulsive.[(12)]

We note that the above arguments regarding self-consistency need not hold for resonant solutions ($(s_R)_{in} > (B+P)^2$), since the representation of the partial wave amplitude by the fixed energy dispersion relation is not expected to hold in the physical region (In other words we cannot directly use Eq. (A.2) to judge the self-consistency in $\mathcal{K}$.). From this one might guess that the self-consistency may get worse as one goes sufficiently above the physical threshold $((s_R)_{in} > (B+P)^2 + 50\ m_\pi^2$, say). This is found to be the case by actual calculation in various systems.

REFERENCES AND FOOTNOTES

1. G. L. Kane, Phys. Rev. 135, B843 (1964). J. C. Pati (unpublished). In this work the (Σ Λ)-mass difference is retained in contrast to Kane's work.

2. J. C. Pati, Phys. Rev. 134, B387 (1964).

3. M. Der Sarkissian, Nuovo Cimento 30, 894 (1963).

4. L. A. P. Baláz̈s, Phys. Rev. 126, 1220 (1962).

5. K. Vasavada (to be published).

6. By "effective" we mean the combined effect of the Σ and Λ terms.

7. It is hard to investigate what happens below this range because of the singularity structure.

8. G. L. Kane, Phys. Rev. 135, B843 (1964). The $I = 1$, $\bar{K}\,\Xi$ system is also considered in this paper. But it drops the (Σ Λ)-mass difference, which leads to some quantitative discrepancy between the results of Kane and that of ours.

9. Y. L. Pan and R. P. Ely, Phys. Rev. Letters 13, 277 (1964).

10. L. Lyons and O. I. Dahl, Physics Letters 14, 255 (1965). Earlier references may be found here.

11. V. Singh and B. M. Udgaonkar, Phys. Rev. 130, 1117 (1963).

12. Even if one considers a system with repulsive Born terms it is quite possible to obtain a self-consistent bound state (or resonant) solution in the Baláz̈s type procedure. For example we tried just as a test negative values of $g^2_{Y\,\Xi\,K}$ (i.e., $g^2/4\pi \simeq -4$, say) in the present calculation and still obtained self-consistent solution. Kane (Reference 1) also noted a similar situation. Of course, for large negative values of $g^2_{Y\,\Xi\,K}/4\pi$ it is not possible to obtain self-consistent solution.

13. We have not checked yet if similar situation holds for other values of angular momentum and parity.

14. In view of the role played by the nearby singularities ($N_{(n)}$) and $g_{1^+}^{(Born)}$ and the fact that the self-consistency cannot quite rigorously be judged on the basis of Eq. (A.2), it is quite possible to obtain a low lying (low lying compared to the physical threshold $(B+P)^2$) resonance rather than a bound state solution. This is what happens in case of (3,3) πN resonance,[11] for example.